CALCULATION OF SEPARATED FLOWS WITHIN THE FRAMEWORK OF THE MODEL OF AN IDEAL FLUID WITH ALLOWANCE FOR A TURBULENT SHEAR LAYER ON THE BOUNDARY OF THE SEPARATION REGION

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Using the example of the problem of a supersonic flow about bodies with a forward separation zone, the authors propose a theoretical model based on the model of an ideal fluid with allowance for a turbulent shear layer on the boundary of the separation region.

The concept of an ideal fluid has become more widely used in recent years in the modeling of supersonic flow about solids of revolution with maximal Reynolds numbers, including cases of separated flow. According to [1-5], in the solution of nonsteady Eulerian equations by finite-difference methods, the wave mechanism of perturbation transfer plays a large role in the formation of circulating flows. The solution of the problem is also significantly affected by artificial viscosity due to errors of the difference approximation of the initial differential equations. In principle, when reproducing flows within the framework of the ideal fluid model, it can be assumed that the wave mechanism of formation of circulating flows is the predominant mechanism and that the effect of artificial viscosity should be reduced to a minimum. Such an approach is reflected in the development of adaptive algorithms based on Godunov's nonsteady difference scheme in conjunction with movable computational grids consistent with gasdynamic features of the flow discovered during the calculation (see [1-3], for example). The scheme was used in [2] to calculate separated flow of a supersonic stream about a blunt body with a projecting conical nozzle. Here, a tangential discontinuity was shed from the sharp edge of the nozzle. However, it should be noted that in this case there are serious restrictions on the exchange of momentum and energy between the external flow and the flow in the circulation zone.

An alternative approach to the calculation of flow about bodies under conditions of flow separation was developed in [4, 5]. This approach is based on the use of the artificial viscosity inherent in difference schemes and affecting the turbulent transport properties seen experimentally. Problems are solved using through-count difference methods (the Godunov method and the method of "fluid particles in a cell") in combination with stationary grids. As was shown in [5] in the example of the calculation of supersonic flow about a cylinder with a projecting disk-shaped nozzle, preference cannot be given to any of the methods mentioned (the wave resistance coefficient is nearly independent of the choice of theoretical method). A study made of the effect of the mesh of the grid on the calculated results showed that the results were close on a sequence of transformed rectangular grids. This permitted the conclusion that the solutions thus obtained are unique, and thus basically resolves the question of the adequacy of the mechanism of physical diffusion and artificial diffusion due to artificial viscosity.

Here, we compare the above-mentioned methods of calculating supersonic flow about bodies under conditions of flow separation (Fig. 1a) within the framework of the ideal fluid model. We attempt to evaluate the adequacy of this approximate approach by comparing theoretical and experimental results. Finally, we give a detailed description of the hybrid method proposed in [6]. This method allows for the presence of a turbulent shear layer on the boundary of the separation region.

The effect of artificial viscosity on the solution of problems for nonseparated flows has been fairly well studied (see [7], for example). It ensures stable calculation of the nonremovable discontinuities which arise in supersonic flows without complements to stabilize the system of Eulerian equations. As a result. it is possible to obtain a monotonic

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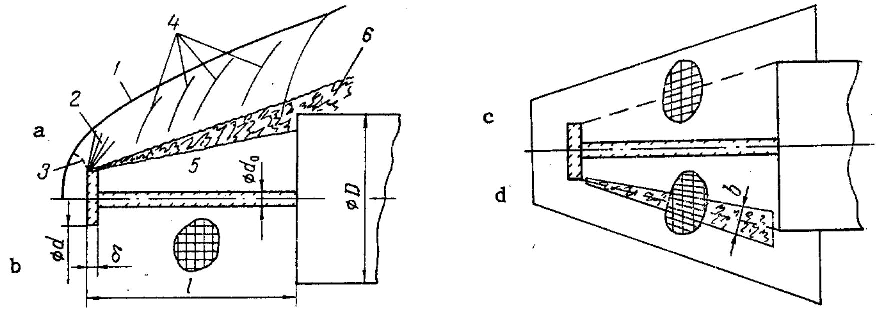

Fig. 1. Physical pattern of flow about a cylinder with a cylinder with a disk (a) (the main structural elements: 1) leading shock wave; 2) rarefaction sheafs; 3) Mach line; 4) compression waves; 5) separation zone; 6) turbulent shear layer) and theoretical regions with a rectangular grid (b), an oblique grid (c), and an oblique grid with a superimposed turbulent shear layer (d).

solution in the neighborhood of shock waves and to determine contact and tangential discontinuities. However, the approach at the same time leads to the erosion of these surfaces in several cells of the computing grid. As was shown by test calculations of model unidimensional problems in [7], the width of the transition region and the maximum gradient of the solution in this region are determined mainly by the dependence of the artificial viscosity coefficient on the mesh of the grid and the transport velocity. Contact discontinuities are particularly eroded.

More complicated and less studied is the question of the effect of artificial viscosity on the solution of two-dimensional problems of supersonic flow about bodies under flow separation conditions. Analytical and numerical studies of laminar and turbulent separated flows of an incompressible fluid by means of first-order difference methods of approximation on rectangular grids indicate the presence of an artificial viscosity which is specific to two-dimensional problems and is connected with slanting of the flow relative to the boundaries of the grid cells. Experience in the solution of various problems has shown that the artificial viscosity introduced into regions of large gradients is due mainly to errors in discretizing the convection terms of the Navier–Stokes equations (or Reynolds equations). In the case of large Reynolds numbers, this viscosity actually overshadows the effect of actual transport processes and seriously distorts the flow pattern [8-10]. Good agreement was obtained between relations derived for the artificial viscosity coefficient V_f on the basis of numerical experiments for two-dimensional shear flows on a uniform grid with a mesh Δ in a modeling of circulating flow in a square cavity induced by the motion of one of the boundaries (see [9], for example) and from the analysis in [8] of the first differential approximation by means of a coordinate rotation. Following [8], we write

$$\nu_f = \frac{\sqrt{2}}{4}\sin\left(\frac{\pi}{4} + \theta\right) q\Delta \sin(2\theta). \quad (1)$$

The maximum values of artificial viscosity are obtained with convective transport of the determining quantities along a diagonal of the computing grid. Thus, artificial diffusion can be reduced significantly in calculations by reorienting he cells along the direction of the velocity vector in regions of large gradients of the flow parameters – especially in the shear layers.

Two types of grids were used in the numerical studies to model supersonic flow about a cylinder of diameter D with a disk nozzle (Fig. 1b) having the geometric dimensions: d = 0.23, δ = 0.07, d_0 = 0.1 (all of the dimensions are given in percents of the cylinder diameter; as the characteristic values in normalizing the gasdynamic parameters, we chose the density and velocity of the undisturbed flow). The Mach number M_∞ of the undisturbed flow was given a value of 4.15, while the projection of the disk ℓ ranged from 0.9 to 1.8. The rectangular grid shown in Fig. 1b was used to model the flow in [4, 5] and in several other studies. It follows from (1) that the shear layer which develops on the boundary of the circulating flow with the external flow is to a considerable extent due to artificial viscosity. A grid with oblique cells (Fig. 1c) was constructed so that the longitudinal borders of the cells in the shear-flow region would be oriented parallel to the velocity vector of the flow as

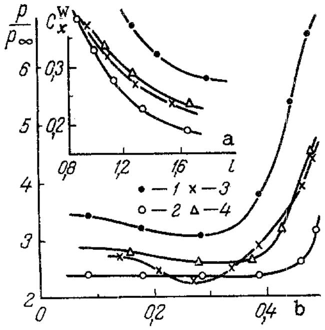

Fig. 2

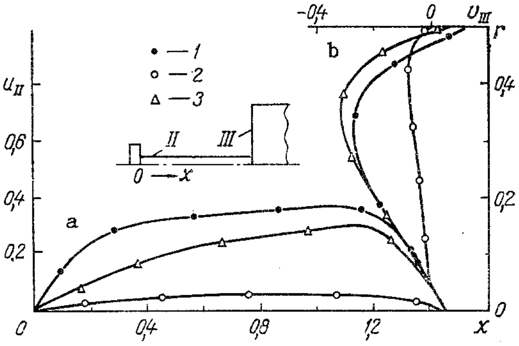

Fig. 3

Fig. 2. Dependence of the coefficient of wave resistance C_x^W of the body on the projection of the disk ℓ (a) and profiles of static pressure on the end surface of the cylinder for ℓ = 1.45 (b): 1, 2) calculations on rectangular and oblique grids; 3) experiment; 4) calculation with a superimposed shear layer.

Fig. 3. Distributions of the axial component of velocity u on the connector between the disk and cylinder (a) and of the radial velocity component v on the front end of the cylinder (b) with ℓ = 1.45; 1, 2) calculations on rectangular and oblique grids; 3) calculation with a superimposed shear layer.

much as possible and, thus, the effect of artificial viscosity would be reduced. It should be noted that in the last case the upper boundary of the theoretical region is inclined toward the symmetry axis in order to alleviate erosion of the curvilinear shock wave. Calculations were performed on the basis of Godunov's nonsteady finite-difference scheme [3]. The boundary conditions were formulated in the usual manner [4, 5]. The dimensions of the theoretical region in both cases were chosen so that an undisturbed flow existed on the top boundary.

We chose a body configuration with a projecting disk with ℓ = 1.45 as the base variant for comparison of the theoretical and experimental results. For a nonuniform rectangular grid on the disk, the connector, and the front surface of the cylinder, we provided 8, 26, and 17 cells, respectively. The minimum and maximum meshes of the grid were 0.05 and 0.085 in the axial direction and 0.014 and 0.025 in the radial direction. There were 16, 26, and 16 cells, respectively, on the oblique grid on the disk,, connector, and front end of the surface. The minimum and maximum meshes were 0.03 and 0.08 in the axial direction and 0.015 and 0.0325 in the radial direction (on the front end of the cylinder). The number of time intervals until establishment of the flow, was on the order of 1500-2500. The flow about the body was considered to be established when the determining parameters of the flow began to change only by small, prescribed amounts. Using the same variant, we measured static pressure on the front of the cylinder at M_∞ = 4.15 (the Reynolds number Re, determined from the velocity of the undisturbed flow and the cylinder diameter, was $1.6 \cdot 10^6$). We also conducted gravimetric tests of the model to determine the dependence of the coefficient of wave resistance C_x^W of the body (normalized over the cross-sectional area of the cylinder) on the projection of the disk nozzle ℓ [6].

Figures 2-4 show some of the results obtained.

It turns out that the solution of the problem depends appreciably on the type of computing grid used. The substantial difference between the coefficients of wave resistance (on the order of 30-40%), profiles of static pressure, and axial and radial velocity components with the change from the oblique grid to the rectangular grid is connected with the difference in the amount of artificial viscosity (see Fig. 4b) realized when the grids are used. For example, as was shown in [5], the difference solution of the problem of supersonic flow about bodies with a forward separation zone within the framework of the ideal fluid model does not depend on the initial conditions of the problem if it is obtained by

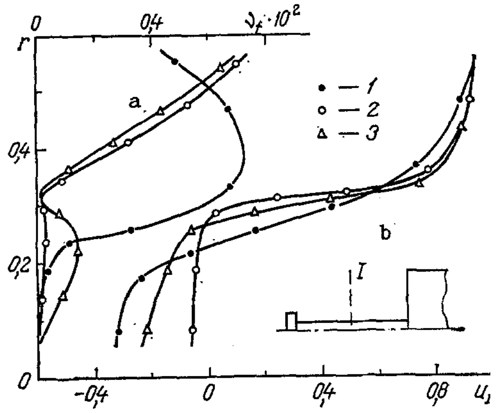

Fig. 4. Profiles of artificial viscosity ν_f (a) and the axial velocity component u (b) in a section perpendicular to the symmetry axis and located a distance x = 0.69 from the projecting disk for ℓ = 1.45: 1, 2) calculations on rectangular and oblique grids; 3) calculation with a superimposed shear layer.

the establishment method. Thus, viscous effects play a very significant role in the formation of the steady circulating flow.

The results shown, together with data from methodological investigations (see [5], for example), permit the conclusion that the type of grid has a greater effect on the solution than does a significant change in the mesh of the grid. It is evidently possible to obtain different solutions with the construction of different grids having the same (limited) number of cells. Thus, the reliability of the solution of the problem with the above-examined idealized formulation is not so apparent. In this connection, it is interesting to note that the methods analyzed here for constructing grids are to a certain degree limiting for the problem being studied, since minimal structural viscosity is introduced with the oblique grid and maximal artificial viscosity is introduced with the rectangular grid (Fig. 4a). We should also point out that the experimental values of the coefficient C_x^w and the static pressure lie between the values calculated on both grids.

On the whole, the results obtained confirmed the original presumption regarding the decisive effect of the shear layer on the circulating flow. In calculations on the oblique grid, the nearly complete exclusion of artificial viscosity in the region of mixing of the flows on the boundary between the circulating flow and the external flow leads to sudden decay of the flow in the region between the disk and cylinder (Figs. 3 and 4). In this case, the flow in the circulation zone is practically isobaric, except for a small neighborhood about the point of attachment of the dividing line of the flow. Here, a local pressure maximum is realized (see Fig. 2b). In this regard, the solution obtained on the oblique grid is similar to solutions obtained with the use of adaptive algorithms and in essence describes the flow of an ideal fluid with a tangential discontinuity (see Fig. 4b) and a nearly stagnant region between the disk and cylinder. The coefficient of wave resistance of the body and the pressure profile on the end surface of the cylinder in the calculations on this grid are below the experimental values, which indicates that turbulent transport in the shear layer has an effect on the distribution of local loads on the body and its total drag.

Calculations of supersonic flow about the body performed on the rectangular grid showed the presence of intensive diffusive transport in the region of the shear layer, which leads to the formation of a developed circulation zone with a maximum flow velocity on the order of 30-40% of the velocity of the undisturbed flow (Fig. 3). The solution is characterized by strong erosion of the shear layer (Fig. 4b), displacement of the point of attachment of the dividing line of the flow toward the symmetry axis relative to the position of this point in the calculations on the oblique grid, and mechanical loads on the end of the cylinder and the body as a whole, which are higher than the experimental loads (Fig. 2a and b). Thus, despite the fact that the suppression of artificial viscosity leads to qualitative similitude of the modeled flow and its physical analog (note the similitude of the pressure profiles in Fig. 2b), the quantitative disagreement of the theoretical and experimental results indicates the inadequacy of the mechanisms of artificial and eddy diffusion.

It follows from the above that difference modeling of supersonic flow about a cylinder with a projecting disk on the basis of the system of nonsteady Eulerian equations does not provide a unique solution on the grids which can be used with modern computers. The modeling fails to do so because of the unsatisfactory reproduction of the calculation of circulating flow between the disk and cylinder. Turbulent transfer of momentum and energy in the shear layer which develops on the mixing boundary of flows with different physical properties plays the deciding role in the formation of circulating flows of the type examined here. Thus, to describe this or a similar flow with circulation zones within the framework of the ideal fluid model corresponding to maximum Reynolds numbers ($Re \to \infty$), it is necessary to at least consider the features inherent to turbulent flow in the shear layer. It seems logical that such a "hybrid" approach to the solution of the stated problem could be realized by isolating the shear layer during the solution and examining turbulent fluid motion in the layer. Here, we combine the solutions inside the shear layer and outside it, where the above algorithms are used.

Turbulent motion in the shear layer can be described by the system of Reynolds equations closed by means of some turbulence model. The effort to construct sufficiently simple and relatively convenient algorithms stimulated us to select a semiempirical convective Prandtl model of turbulence. Calculation of separated turbulent flows of an incompressible fluid at high Reynolds numbers (see [10], for example) showed that in a region with large parameter gradients, it is best to reduce artificial viscosity either by increasing the order of the difference scheme approximation or by using computing grids which are adaptive to the flow features. Thus, in solving the stated problem of flow about a cylinder with a projecting disk, we used an oblique grid with a reference line which joined the sharp edges of the disk and cylinder (see Fig. 1d). We also condensed the grid around the reference line.

Within the limits of an a priori prescribed shear layer, the Reynolds equations in cylindrical coordinates x, φ, and r are represented in generalized form similar to the Eulerian equations. The Reynolds equations have additional terms describing turbulent transfer of momentum and energy and energy dissipation:

$$\frac{\partial\sigma}{\partial t} + \frac{\partial A}{\partial x} + \frac{\partial B}{\partial r} + \frac{F}{r} = 0, \tag{2}$$

where

$$\sigma = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{Bmatrix}; \; A = \begin{Bmatrix} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho uv - \tau_{xr} \\ (e+p)u - u\tau_{xx} - v\tau_{xr} - Q_x \end{Bmatrix};$$

$$B = \begin{Bmatrix} \rho v \\ \rho uv - \tau_{xr} \\ \rho v^2 + p - \tau_{rr} \\ (e+p)v - u\tau_{xr} - v\tau_{rr} - Q_r \end{Bmatrix}; \; F = \begin{Bmatrix} \rho v \\ \rho uv - \tau_{xr} \\ \rho v^2 - \tau_{rr} - \tau_{\varphi\varphi} \\ (e+p)v - v\tau_{rr} - u\tau_{xr} - Q_r \end{Bmatrix}.$$

Here $\tau_{xx} = 2\rho\nu_t \frac{\partial u}{\partial x}$; $\tau_{xr} = \rho\nu_t\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)$; $\tau_{rr} = 2\rho\nu_t \frac{\partial v}{\partial r}$; $\tau_{\varphi\varphi} = 2\rho\nu_t \frac{v}{r}$; $Q_x = \frac{\rho\nu_t}{\Pr_t}\frac{\partial T}{\partial x}$;

$$Q_r = \frac{\rho\nu_t}{\Pr_t}\frac{\partial T}{\partial r}; \; q^2 = u^2 + v^2; \; e = \frac{p}{\varkappa - 1} + \frac{\rho q^2}{2}; \; \varkappa = \frac{C_p}{C_v} = 1.4; \; \Pr_t = 0.9.$$

In keeping with the adopted Prandtl turbulence model, the eddy viscosity coefficient is determined in the form

$$\nu_t = C_t\,(u_{\max} - u_{\min})\,b, \tag{3}$$

where u_{max} and u_{min} are the flow velocities on the boundaries of the shear layer; C_t is an empirical constant; b is the thickness of the layer. The thickness of the layer is calculated in accordance with the similarity solution [11]

$$b = C_n s, \tag{4}$$

where C_n is an empirical constant; s is the coordinate reckoned along the reference line.

The eddy viscosity coefficient is assumed to be constant over the thickness of the shear layer, so that the components of the turbulent friction tensor undergo a discontinuity on the boundaries of the layer first stage of the study entails introduction of the presumption of symmetrical development of the shear layer along the reference line. The turbulence constant is selected on the basis of recommendations made in several investigations (see [11, 12], for example) and is checked during the study by comparing theoretical and experimental results. Also, the correctness of the choice of the constant C_n needed to determine the thickness of the shear layer b is evaluated indirectly by measuring the thickness of the layer above the reference line using data from optical visualization of the flow. The range of the constant was determined by investigation: $C_n = 0.08-0.13$; $C_t = 0.01-0.015$. Most of the calculations were performed with the values $C_n = 0.085$ and $C_t = 0.015$.

Figures 2-4 show results of calculations performed by the method described above. We should point out not only the good agreement between the theoretical and experimental results on the coefficient of wave resistance and the static pressure distribution on the end surface of the cylinder (Fig. 2), but also the fairly high velocity of circulating flow. The velocity reaches 30% of the velocity of the undisturbed flow. It should be noted that use of the shear model does not significantly change the external flow and has almost no effect on the position of the point of attachment of the flow on the forward end of the cylinder (Fig. 3).

Thus, the "hybrid" approach proposed here for the solution of problems of separated flow of an ideal fluid about bodies with allowance for the turbulent shear layer developing on the boundary of the separation region makes it possible to expand the range of application of the adaptive algorithms devised in [1-3]; the reliable prediction achieved here for local and integral mechanical characteristics for the above-examined special problem of supersonic flow about a cylinder with a projecting disk nozzle indicates that it is possible to generalize the approach to the class of problems of flow (subsonic and supersonic) about bodies of fairly arbitrary shape. This includes the problem of calculating flow in the near wake behind the body.

NOTATION

x, ϕ, r, axial, circumferential, and radial coordinates, respectively; u, v, axial and radial components of velocity; q, modulus of flow velocity vector; θ, angle between flow velocity vector and symmetry axis; ρ, density of the gas; ν, kinematic viscosity coefficient; T, temperature; p, static pressure; d, diameter of disk; ℓ, projection of disk ahead of cylinder; δ, thickness of disk; d_0, diameter of connector; D, diameter of cylinder; Δ, mesh of grid; M, Mach number; Re, Reynolds number; C_x^W, coefficient of wave resistance; C_p, C_V, heat capacities at constant pressure and volume, respectively; Pr_t, turbulent Prandtl number. The indices: ∞, parameters of the undisturbed flow; T, turbulent characteristics; f, parameters characterizing artificial diffusion.

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